

Comparison of Non-Linear Functions to Describe the Growth in Mengali Sheep Breed of Balochistan

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Abstract.- The objectives of this study were to estimate and compare the goodness of fit six non-linear growth models, *i.e.* Gompertz, Logistic, Von Bertalanffy, Richards, Weibull and Morgan-Mercer-Flodin (MMF), in Mengali sheep of Balochistan. For this purpose, monthly body weight data from 2377 sheep from birth up to 360 days of age were used. The average body weights in each period were used to define the weight – age relationship in these sheep. The models parameters, adjusted coefficient of determination (R^2_{Adj}), root mean square error (RMSE), Akaike's and Bayesian information criteria (AIC and BIC), predicted body weights and residuals were calculated for each model. Among six growth functions, the MMF model was found appropriate for its accuracy of fit according to the highest R^2_{Adj} (0.9991) and the lowest RMSE (0.3348) followed by the Von Bertalanffy and Weibull models. The MMF model was also selected the best based on the lowest values of AIC (9.6516) and BIC (10.0489). The results of this study suggest that the MMF model can be used to accurately predict the growth in body weight of Mengali sheep.

Key words: Mengali sheep, weight-age relationship, growth models, non-linear regression models

INTRODUCTION

Among many different breeds of sheep in Balochistan, the Mengali sheep is one of the important breed found in black or brown color with white patches on the body, prominent head with black face but white forehead (Khan *et al.*, 2007). According to Kakar and Ahmad (2004) the mature body weight is 34 kg in the rams and 27 kg in the ewes. Though the source of origin of Mengali sheep is not known, it is mostly raised by native “Mengal” tribe (most populated tribe of Kalat Division area) therefore the breed is famous as Mengali (local farmers also name it *Budi*) Khadkucha (Mastung) is identified to be a hub of this breed (Tariq *et al.*, 2011).

Growth is one of the most important traits in farm animals and defined as an increase in body size per unit time (Topal *et al.*, 2004). Body growth is related to an increase in cell number and volume.

Age related changes can be observed in the weight or size of any organ, in the composition of tissues, in cell size and number as well as in live weight (Eisen, 1976). Growth curves are also used for investigating optimum feeding programmes, determining optimum slaughtering age and the effects of selection on curve parameters and on live weight at a certain age (Blasco and Gomes, 1993).

Among statistical procedures available for analyzing growth data, fitting a non-linear function offers an opportunity to summarize the information contained in the entire sequence of size–age points into a small set of parameters that can be interpreted biologically and used to derive other growth traits (Perotto *et al.*, 1992). Numerous growth equations have been developed to describe and fit the nonlinear sigmoid relationship between growth and time. Changes in live weight or dimension for a period of time are explained by the growth curves such as, Brody, Gompertz, Logistic and Von Bertalanffy models in Turkish Kivircik and Daglic male lambs (Akbas *et al.*, 1999), in Awassi lambs (Tekel *et al.*, 2005), in Akkaraman and German black headed mutton x Akkaraman (Kucuk and

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Eyduran, 2009) and in Mengali sheep (Tariq *et al.*, 2011).

The objectives of this study were to compare the goodness of fit of non-linear function (Gompertz, Logistic, Von Bertalanffy, Richards, Weibull and Morgan-Mercer-Flodin) and to provide a specific shape of the growth curve according to the environmental effects on monthly weight from birth to 360 days of age in Mengali sheep.

MATERIALS AND METHODS

The pedigree and performance data on Mengali sheep of Balochistan recorded on 2377 lambs descended from 581 ewes and 56 rams in four flocks at three different locations were available for the present study. Data on four sheep flocks maintained at three places Mastung (Khadkocha), Nushki (Peer Wala) and Quetta (Killi Hassni) and Experimental station at Centre for Advanced Studies in Vaccinology and Biotechnology (CASVAB) (ESC), University of Balochistan (UoB) Brewery Road, Quetta were collected monthly from birth to 360 days of age during Jan 2005 to Dec 2009. Table I shows the average body weight of these sheep for each period. The same data set was used by Tariq *et al.* (2011) and the Gompertz curve was fitted to the Mengali sheep data.

Table I.- Average body weight of Mengali sheep by age.

Age (days)	Weight (kg)
0	3.62
30	8.37
60	12.61
90	16.74
120	20.46
180	25.81
270	31.13
360	36.03

Following fixed effect models were assumed: Model I for birth and weaning weight while Model II for other traits.

$$Y_{ijklmn} = \mu + F_i + YOB_j + SOB_k + TOB_l + SEX_m + b(P_{ijklmn}) + e_{ijklmn} \quad \text{(Model I)}$$

$$Y_{ijklmn} = \mu + F_i + YOB_j + SOB_k + TOB_l + SEX_m + e_{ijklmn} \quad \text{(Model II)}$$

Where; Y_{ijklm} = Trait of interest observed; μ = Population mean; F_i = Effect of the flock (1-4); YOB_j = Year of birth; SOB_k = Season of birth (Spring, Autumn); TOB_l = Type of birth (Single, twin); SEX_m = Male and female; b = regression coefficient; P_{ijklmn} = maternal effect of dam, e_{ijklmn} = random error associated with each observation. It is assumed to be normally distributed with mean zero and variance σ^2 .

The model for growth curves is the equations which describe the increase of a body weight against age. These models differ according to the number of parameters used in the equation being calculated by the measured data. There are many non-linear models that have been used to model growth of domestic animals. The most in use are three-parameter growth functions (Gompertz, logistic and Von Bertalanffy) and four-parameter equations (Richards, Weibull and Morgan-Mercer-Flodin). In the present study, these six widely used non-linear growth models were retained to describe the evolution of body weight of Mengali sheep with time. The mathematical relations of these models are as follows:

Gompertz:
$$W_t = A \exp(-\exp(-k(t-l))),$$

Logistic:
$$W_t = A \frac{\exp(-k(t-l))}{1 + \exp(-k(t-l))},$$

Von Bertalanffy:
$$W_t = A (1 - \exp(-k(t-l))),$$

Richards:
$$W_t = A [1 + (\delta - 1) \exp(-k(t-l))]^{\frac{1}{1-\delta}},$$

Weibull:
$$W_t = A - (A - B) \exp(-(kt)^\delta),$$

Morgan-Mercer-Flodin:
$$W_t = A - \frac{A - B}{1 + (kt)^\delta},$$

where W_t is the live body weight of Mengali sheep at time t ; A is the asymptotic weight when age approaches infinity indicating the average weight of mature sheep; k is the maturing rate; l is the ordinate of the inflection point; δ is the parameter that controls the point of inflection and B is the

lower asymptote.

The comparison amongst the models were based on the coefficient of determination (R^2) which shows how well a model fits the data and the root mean square error ($RMSE$) which measures the error in squared terms. The adjusted coefficient of determination (R^2_{Adj}) was also used for evaluation as the number of parameters in models is different. In addition, two model selection criteria namely Akaike's information criteria (AIC) and Bayesian information criteria (BIC) were calculated. The best model will have the highest value of R^2_{Adj} and the lowest values for $RMSE$, AIC and BIC.

The data were analyzed using Matlab[®] 2012 software. The non-linear least squares algorithm of Levenberg-Marquardt with convergence criteria of $1.0E-08$ was used for the estimation of the parameters of various models.

RESULTS AND DISCUSSION

Table II represents the estimated parameters along with standard errors of different growth models. All parameters are found significant ($P < 0.05$) except for two parameters of Richards model (l and k). Hence, we exclude the Richards model for further evaluation.

Table III shows the predicted values and residuals obtained after fitting each model to the body growth data. The goodness of fit measures (R^2 , R^2_{Adj} , $RMSE$, AIC and BIC) are also displayed. The determination coefficient for the MMF model was found the highest (0.9995) and the lowest for Logistic model (0.9943). The values for Bertalanffy and Weibull models were similar (0.9993) followed by the Gompertz model (0.9943). The adjusted coefficient of determination, in descending order, for MMF, Bertalanffy, Weibull, Gompertz and Logistic models were 0.9991, 0.9990, 0.9987, 0.9921, and 0.9792, respectively. The MMF, Bertalanffy and Weibull models showed the highest values.

When models are evaluated based on $RMSE$, the MMF model showed the lowest values (0.3348) followed by Bertalanffy (0.3577), Weibull (0.3988), Gompertz (0.9994) and Logistic (1.6167). The MMF model outperformed other competing models according to goodness of fit by $RMSE$. This model

also showed the minimum AIC and BIC values (AIC=9.6516, BIC=10.0489) followed by Bertalanffy (AIC=10.4654, BIC=10.8132) and Weibull (AIC=12.4501, BIC=12.8473). The values of AIC and BIC we found very high for Gompertz (AIC=26.9336, BIC=27.2514) and Logistic model (AIC=34.6289, BIC=34.9467).

Based on various evaluation measures we found that the MMF model has the best fitting to the body weight data of Mengali sheep. The Bertalanffy and Weibull models showed higher accuracy. The performance of Gompertz model had smaller R^2_{Adj} and larger $RMSE$ and AIC/BIC as compared to the three models mentioned above. The Logistic model had the least performance as compared to other non-linear growth models used in this study showing least values for adjusted coefficient of determination and the highest value for root mean square error, AIC and BIC.

The rank of the model was determined based on three evaluation criteria (R^2_{Adj} , $RMSE$, AIC, BIC) and showed as a superscript on models in Table III. We can say that the best three models for describing the growth rate of Mengali sheep are Morgan-Mercer-Flodin, Von Bertalanffy and Weibull model, in descending order.

The Brody function was found to be sufficient for the growth in Morkaraman, Awassi and Tushin sheep by Esenbuğa *et al.* (2000) and Bilgin and Esenbuga (2003). The Gompertz function was found to be appropriate for describing the growth curve of Suffolk sheep (Lewis *et al.*, 2002). The Gompertz and Bertalanffy were found the best fitted functions for the Morkaraman and Awassi breed, respectively, by Topal *et al.* (2004). On the other hand, Tekel *et al.* (2005) concluded that Logistic, Gompertz and Bertalanffy models described growth of Awassi lambs better than Brody and Negative exponential models.

For Kivircik breed, the reported value of R^2 was 0.993 (Akbas *et al.*, 1999), for Morkaraman, Awassi and Tushin lambs were 0.99, 0.99 and 0.98, respectively (Esenbuğa *et al.*, 2000) for Awassi lamb the reported values was 0.98 (Tekel *et al.*, 2005) and for Norduz female lamb it was 0.997 (Kum *et al.*, 2010). Tariq *et al.* (2011) fitted the Gompertz curve to the same Mengali sheep data and the reported values of R^2 and $RMSE$ were 0.992

and 1.022, respectively. They concluded that the

Table II.-Estimated parameters±SE of various growth models fitted to body weight.

Model	Parameters				
	A	k	l		B
Gompertz	36.966±1.4049	0.0100±0.0010	70.8299±5.6480	–	–
Logistic	35.0583±1.6312	0.0156±0.0022	103.2866±10.0145	–	–
Von Bertalanffy	44.2992±1.2281	0.0044±0.0003	-18.9810±2.1265	–	–
Richards	44.0385±2.9052	0.0044±0.0010	-16.0245±29.8712*	0.0158±0.1623*	–
Weibull	43.8946±2.9209	0.0044±0.0006	–	1.0092±0.0620	3.5383±0.3892
Morgan-Mercer-Flodin	57.0618±4.6581	0.0040±0.0006	–	1.1052±0.0707	3.5992±0.3289

A is the asymptotic weight when age approaches infinity indicating the average weight of mature sheep; k is the maturing rate; l is the -ordinate of the inflection point; is the parameter that controls the point of inflection and B is the lower asymptote. *Not significant at 10%. All other parameters are significant (P<0.05).

Table III:- Goodness of fit measures for various growth models fitted to the body weight data.

Age	Observed Weight	Gompertz ⁴		Logistic ⁵		Bertalanffy ²		Richards		Weibull ³		MMF ¹	
		P	R	P	R	P	R	P	R	P	R	P	R
0	3.62	4.83	-1.21	5.82	-2.20	3.51	0.11	3.53	0.09	3.54	0.08	3.60	0.02
30	8.37	8.20	0.17	8.46	-0.09	8.49	-0.12	8.48	-0.11	8.47	-0.10	8.33	0.04
60	12.61	12.12	0.49	11.82	0.79	12.87	-0.26	12.86	-0.25	12.85	-0.24	12.83	-0.22
90	16.74	16.20	0.54	15.72	1.02	16.71	0.03	16.71	0.03	16.71	0.03	16.77	-0.03
120	20.46	20.07	0.39	19.81	0.65	20.09	0.37	20.09	0.37	20.09	0.37	20.17	0.29
180	25.81	26.46	-0.65	26.93	-1.12	25.64	0.17	25.66	0.15	25.69	0.14	25.67	0.14
270	31.13	32.28	-1.15	32.65	-1.15	31.68	-0.55	31.70	-0.57	31.70	-0.57	31.61	-0.48
360	36.03	34.99	1.04	34.43	1.60	35.77	0.26	35.75	0.28	35.75	0.28	35.78	0.25

Goodness of fit		Gompertz ⁴		Logistic ⁵		Bertalanffy ²		Richards		Weibull ³		MMF ¹	
R ²		0.9943		0.9852		0.9993		0.9993		0.9993		0.9995	
R ² _{Adj}		0.9921		0.9792		0.9990		0.9987		0.9987		0.9991	
RMSE		0.9994		1.6167		0.3577		0.3995		0.3988		0.3348	
AIC		26.9336		34.6289		10.4954		12.4759		12.4501		9.6516	
BIC		27.2514		34.9467		10.8132		12.8731		12.8473		10.0489	

Superscript on model represents the rank of the model; P and R are the predicted weight and residuals, respectively, R²_{Adj} adjusted coefficient of determination, RMSE: root mean square error, AIC and BIC are Akaike's and Bayesian information criteria, respectively.

Gompertz curve reliably explained relationship between weight and age in Mengali sheep. The R² and RMSE values of Gompertz curve in this study are in line with Tariq et al. (2011). In addition we calculated the adjusted determination coefficient and model selection criteria (AIC and BIC). A slight difference in RMSE value may be due to different software we used for estimating non-linear model. The Gompertz curve though fit the data reasonably as mentioned by Tariq et al. (2011) but ranked 4 out of 5 models considered in the present study.

Figure 1 shows the fitted Moran-Mercer-Flodin curve and Gompertz curve against the actual

(observed) values of body weight of Mengali sheep

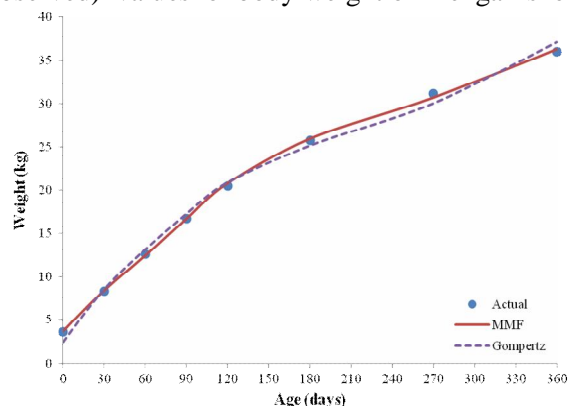


Fig. 1. Morgan-Mercer-Flodin (MMF) and Gompertz growth curve fitted to body weight from birth to 360 days of age. The fitted curves of Von Bertalanffy and Weibull model showed the same trend as MMF and therefore not displayed. The Gompertz curve is plotted in order to compare its difference with that of the best fitted curve (MMF) in this study. It is evident from the figure that the MMF growth curve predicted the body weight of Mengali sheep better than the Gompertz growth curve. This further supports our findings that the MMF model describes the growth of body weight better than the Gompertz model.

In this study, six different growth models (Gompertz, Logistic, Von Bertalanffy, Richards, Weibull and Morgan-Mercer-Flodin) were studied to determine the growth in the body weight of Mengali sheep breed from birth to 1 year of age. The MMF model was found to be the best model based on various performance measures such as adjusted coefficient of determination, root mean square error and model selection criteria (AIC and BIC).

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